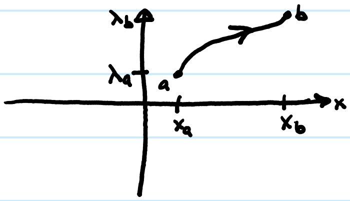
2018-10-29-1

Energy conversion cycles: converting from electrical to mechanical and vise versa. Still a conservative system!!

Recall: $\Delta W_m = \int_{\lambda_a}^{\lambda_b} i\,d\lambda - \int_{x_a}^{x_b} f^e dx \Rightarrow \Delta W_m = \int_{\lambda_a}^{\lambda_b} i\,d\lambda + \left(\int_{x_a}^{x_b} -f^e dx \right)$

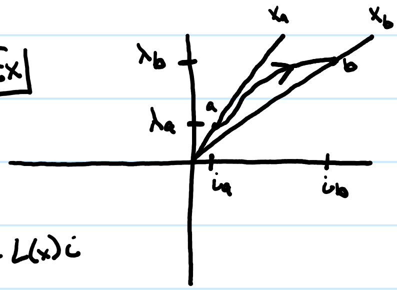


$\Delta W_m = EFE|_{a-b} + EFM|_{a-b}$

EFE: energy from electrical

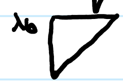
EFM: energy from mechanical

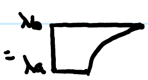
Ex



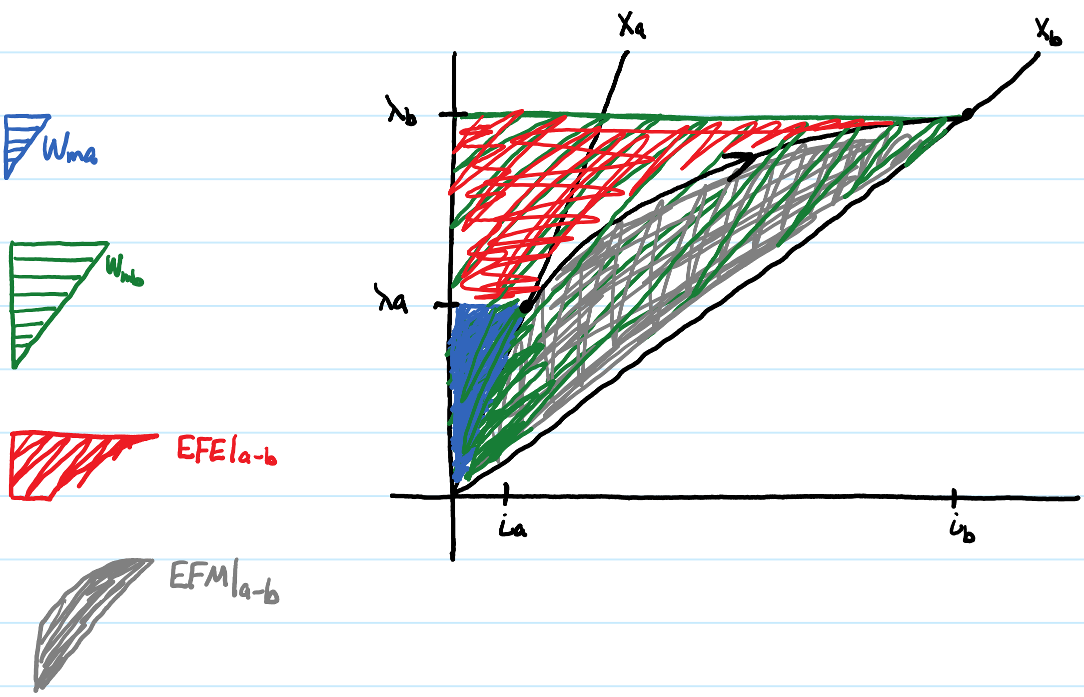
$\lambda = L(x)i$

$W_{ma} = \int_0^{\lambda_a} i(\lambda, x_a)\,d\lambda =$ 

$W_{mb} = \int_0^{\lambda_b} i(\lambda, x_b)\,d\lambda =$ 

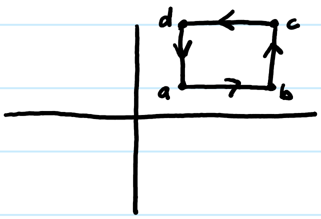
$EFE|_{a-b} = \int_{\lambda_a}^{\lambda_b} i(\lambda, x)\,d\lambda =$ 

$EFM|_{a-b} = (W_{mb} - W_{ma}) - EFE|_{a-b}$



2018-10-29-2

Energy conversion over a cycle:



path: $a \to b \to c \to d \to a$

$\Delta W_m = 0$ (start and end at same point)

$\oint dW_m = \oint i\,d\lambda - \oint f^e dx \Rightarrow \quad 0 = \oint i\,d\lambda + \left(\oint -f^e dx\right)$

$0 = EFE|_{cycle} + EFM|_{cycle}$

* compute either EFE or EFM and use relation to find the other

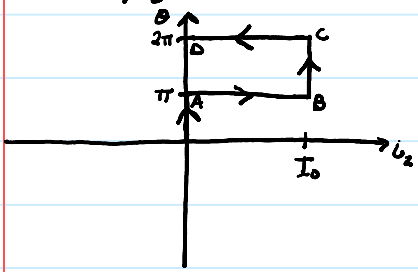
EFE > 0 (EFM < 0): motor

EFE < 0 (EFM > 0): generator

* Flipping cycle direction converts a generator (motor) to a motor (generator)

2018-10-29-3

Ex
$$\lambda_1 = L_1 i_1 + M\cos(\theta) i_2$$
$$\lambda_2 = M\cos(\theta) i_1 + L_2 i_2$$



$i_1 = I_0$ over cycle

Find: EFM. Is this a motor or generator?

Solution: $W_m' = \int_0^{i_1} L_1 \hat{i}_1 \, d\hat{i}_1 + \int_0^{i_2} \left(M\cos\theta \, i_1 + L_2 \hat{i}_2\right) d\hat{i}_2 \Rightarrow W_m' = \frac{1}{2} L_1 i_1^2 + M\cos(\theta) i_1 i_2 + \frac{1}{2} L_2 i_2^2$

$T^e = \frac{\partial W_m'}{\partial \theta} = \quad T^e = -M\sin(\theta) i_1 i_2$

$$EFM|_{cycle} = \int_0^{2\pi} -T^e d\theta = \int_0^{\pi} -T^e d\theta + \int_{\pi}^{2\pi} -T^e d\theta$$

$$= \int_0^{A} M\sin(\theta) i_1 i_2 d\theta + \int_A^{B} M\sin(\theta) i_1 i_2 d\theta + \int_B^{C} M\sin(\theta) i_1 i_2 d\theta + \int_C^{D} M\sin(\theta) i_1 i_2 d\theta$$

$$= \cancelto{0}{\int_0^{\pi} M\sin(\theta) I_0 (0) d\theta} + \cancelto{0}{\int_{\pi}^{\pi} M\sin(\theta) I_0 i_2 d\theta} + \int_{\pi}^{2\pi} M\sin(\theta) I_0^2 d\theta + \cancelto{0}{\int_{2\pi}^{2\pi} M\sin(\theta) I_0 i_2 d\theta}$$

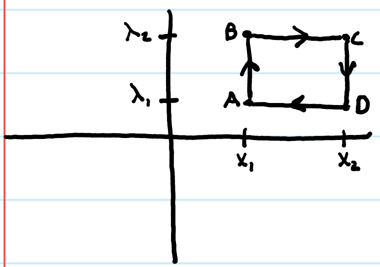
$$= -M\cos(2\pi) I_0^2 + M\cos(\pi) I_0^2$$

$$\boxed{EFM|_{cycle} = -2MI_0^2}$$

$$\boxed{EFM < 0 \Rightarrow \text{motor}}$$

2018-10-29-4

Ex $\lambda = \left(\dfrac{L_0}{1+\frac{x}{a}}\right) i$



Find: a) $f^e(\lambda, x)$

b) $EFM|_{cycle}$

c) Motor or generator?

Solution: a) $i = \left(\dfrac{1+\frac{x}{a}}{L_0}\right)\lambda$

$$W_m = \int_0^{\lambda} i\, d\hat{\lambda} = \int_0^{\lambda} \left(\frac{1+\frac{x}{a}}{L_0}\right)\hat{\lambda}\, d\lambda \Rightarrow W_m = \frac{1}{2}\left(\frac{1+\frac{x}{a}}{L_0}\right)\lambda^2$$

$$f^e = -\frac{\partial W_m}{\partial x} \Rightarrow \boxed{f^e = \frac{-1}{2}\left(\frac{\lambda^2}{aL_0}\right)}$$

b) $$EFM|_{cycle} = \oint -f^e dx = \int_A^B \frac{\lambda^2}{2L_0 a} dx + \int_B^C \frac{\lambda^2}{2L_0 a} dx + \int_C^D \frac{\lambda^2}{2L_0 a} dx + \int_D^A \frac{\lambda^2}{2L_0 a} dx$$

$$= \cancelto{0}{\int_{x_1}^{x_1} \frac{\lambda^2}{2L_0 a} dx} + \int_{x_1}^{x_2} \frac{\lambda_2^2}{2L_0 a} dx + \cancelto{0}{\int_{x_2}^{x_2} \frac{\lambda^2}{2L_0 a} dx} + \int_{x_2}^{x_1} \frac{\lambda_1^2}{2L_0 a} dx$$

$$= \frac{\lambda_2^2}{2L_0 a}(x_2 - x_1) + \frac{\lambda_1^2}{2L_0 a}(x_1 - x_2)$$

$$\boxed{EFM|_{cycle} = \frac{(x_2 - x_1)}{2L_0 a}\left(\lambda_2^2 - \lambda_1^2\right)}$$

c) $x_2 > x_1$ $\qquad \lambda_2 > \lambda_1$

$\boxed{EFM > 0 \Rightarrow \text{generator}}$